

## DEPARTMENT OF HIGHER EDUCATION AND TRAINING REPUBLIC OF SOUTH AFRICA <br> NATIONAL CERTIFICATE <br> MATHEMATICS N3 <br> TIME: 3 HOURS <br> MARKS: 100

## INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but do not separate subsections.
5. Show ALL calculations and intermediate steps.
6. ALL graph work must be done in the ANSWER BOOK. NO graph paper is needed.
7. ALL final answers must be accurately approximated to THREE decimal places.
8. Diagrams are not drawn to scale.
9. Write neatly and legibly.

## QUESTION 1

1.1 Factorise the following expression completely:

$$
\begin{equation*}
x\left(\frac{2 x^{2}}{x}-\frac{3 x y}{x}\right)+y\left(\frac{2 x y}{y}-\frac{3 y^{2}}{y}\right) \tag{3}
\end{equation*}
$$

1.2 When $f(x)=3 a x^{3}+2 x^{2}-7 x+3$ is divided by $2 x+1$ the remainder is $-\frac{a}{2}$.

Determine the value or values of $a$.
1.3 Simplify the following expression:

$$
\begin{equation*}
\frac{b}{a+b}+\frac{a}{a-b}+\frac{2 a b}{b^{2}-a^{2}} \tag{6}
\end{equation*}
$$

## QUESTION 2

2.1 Simplify each of the following:
2.1.1 $\frac{\left(3 x^{2}-2 x-1\right)^{-\frac{3}{2}} \times \sqrt[3]{9 x^{2}+6 x+1}}{\sqrt{(x-1)^{-1}} \times \sqrt[6]{(3 x+1)}}$
2.1.2 $\frac{25^{2 x-1} \times \frac{1}{5}}{5^{3 x+1} \times 5^{x-2}}$
2.2 Solve for $x$ :

$$
\begin{equation*}
\log \left(3^{x}-3^{2-x}\right)=\log 4+\frac{\log 2}{\log _{4} 4} \tag{8}
\end{equation*}
$$

## QUESTION 3

3.1 A taxi driver takes a total of R360 in fares per trip with a full load of passengers. If he increases his fare by R10 per passenger, he can make the same amount when carrying three passengers fewer. Determine:

### 3.1.1 The total increased fare amount

3.1.2 The original number of passengers per trip.
3.2 Make $x$ the subject of the formula:
$\log \left(\frac{a^{x}}{b}\right)=D$
3.3

Solve for $x$ :

$$
\begin{equation*}
\frac{3}{x(x+1)}+\frac{2}{(x+1)(x-2)}=\frac{1}{x(x-2)} \tag{4}
\end{equation*}
$$

## QUESTION 4

Consider FIGURE A. P(-4;6), $\mathrm{Q}(-7,3), \mathrm{R}(1 ;-3)$ and $\mathrm{S}(2 ; 0)$ are the vertices of quadrilateral PQRS.


FIGURE A

Determine:
4.1 The gradient of line PS
4.2 The length of QP
4.3 The equation of the line through $M$, the midpoint of PQ and parallel to PS
4.4 The equation of the line perpendicular to PS passing through the origin
4.5 The equation of the line through $\mathrm{Q}(-7 ; 3)$ with angle of inclination $120^{\circ}$
4.6 The co-ordinates of the point of the intersection of the following two lines given: the line through $\mathrm{Q}(-7 ; 3)$ which is parallel to the y -axis and the line $2 y-5=x$
[17]

## QUESTION 5

5.1 In FIGURE B points A and D are the intercepts of the $x$-axis of $y=-x^{2}+2 x+5$, while point B is the intercept of the $y$-axis and point C the turning point.


FIGURE B
Determine:
5.1.1 The $x$-values of A and D correct to three decimal places.
5.1.2 The coordinates of B and C
5.2 Determine $\frac{d y}{d x}$ by using the rules of differentiation. The final answer must be with a positive exponent and in surd form. The given function is defined as follows:
$y=\frac{2}{\sqrt{x}}+\frac{x^{3}}{3}$
5.3 Consider FIGURE C which represents $y=2 x^{3}+6 x^{2}$. Use differentiation to determine the coordinates of $A$ which is a turning point of the given graph.


FIGURE C

## QUESTION 6

6.1 If $\sin A=\frac{3}{7}$ and $\sec B=\frac{13}{5}$, where $A$ and $B$ are acute angles, determine the value of $40 \tan ^{2} A-12 \cot B$ without using a calculator.
6.2 Use basic trigonometric identities to prove that $\frac{\tan \theta+1}{\sec \theta}=\sin \theta+\cos \theta$.
6.3 In FIGURE D below, AB and CD are two towers that stand on the same horizontal plane. The angle of depression from the top of tower $A B$ to the foot of tower $C D$ is $30^{\circ}$ and the angle of elevation from the foot of tower AB to the top of tower CD is $45^{\circ}$.

If the height of tower $A B$ is $5 \sqrt{3}$, determine the height of tower $C D$.


FIGURE D
6.4 Consider the functions $f(x)=\sin 2 x$ and $g(x)=2 \cos x$.
6.4.1 Draw the graphs of $f(x)$ and $g(x)$ using the same set of axes for $0^{\circ} \leq x \leq 180^{\circ}$. Show the coordinates of the turning points and the intercepts with the axes.
6.4.2 From the graphs find the values of $x$ satisfying each of the following equations:
(a) $\sin 2 x=2 \cos x$
(b) $-2 \cos x=2$

## FORMULA SHEET

Any applicable formula may also be used.

## 1. Factors

$a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)$
$a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right)$

## 3. Quadratic formula

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## 4. Parabola

$$
\begin{aligned}
& y=a x^{2}+b x+c \\
& y=\frac{4 a c-b^{2}}{4 a} \\
& x=\frac{-b}{2 a}
\end{aligned}
$$

## 5. Circle

## 2. Logarithms

$$
\begin{aligned}
& \log a b=\log a+\log b \\
& \log \frac{a}{b}=\log a-\log b \\
& \log _{b} a=\frac{\log _{c} a}{\log _{c} b}
\end{aligned}
$$

$\log a^{m}=m \log a$

$$
\log _{b} a=\frac{1}{\log _{a} b}
$$

$$
\log _{a} a=1 \therefore \ln e=1
$$

$$
a^{\log _{a} t}=t \therefore e^{\ln m}=m
$$

$x^{2}+y^{2}=r^{2}$
$D=\frac{x^{2}}{4 h}+h$
$x=\sqrt{4 D h-4 h^{2}}$

## 6. Straight line

$$
y-y_{l}=m\left(x-x_{l}\right)
$$

Perpendicular: $m_{1} \cdot m_{2}=-1$

Parallel lines: $m_{1}=m_{2}$
Distance: $D=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
Midpoint: $P=\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right)$
Angle of inclination: $\theta=\tan ^{-1} m$

## 7. Differentiation

$$
\frac{d y}{d x}=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}
$$

$$
\frac{d}{d x}\left(x^{n}\right)=n x^{n-1}
$$

Max/Min
For turning points: $f^{\prime}(x)=0$

## 8. Trigonometry

$$
\begin{aligned}
& \sin \theta=\frac{y}{r}=\frac{1}{\operatorname{cosec} \theta} \\
& \cos \theta=\frac{x}{r}=\frac{1}{\sec \theta} \\
& \tan \theta=\frac{y}{x}=\frac{1}{\cot \theta} \\
& \sin ^{2} \theta+\cos ^{2} \theta=1 \\
& 1+\tan ^{2} \theta=\sec ^{2} \theta \\
& 1+\cot ^{2} \theta=\operatorname{cosec}^{2} \theta \\
& \tan \theta=\frac{\sin \theta}{\cos \theta} \\
& \cot \theta=\frac{\cos \theta}{\sin \theta} \\
& \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c} \\
& a^{2}=b^{2}+c^{2}-2 b c \cos A
\end{aligned}
$$

